Algebraic Geometry in Architectural Design

Günter Barczik[1], Oliver Labs[2], Daniel Lordick[3]
[1]Chair of Contextual Design, Dept. of Architecture, Cottbus University, Germany
[2]Institute for Mathematics and Computer Science, University of Saarbrücken, Germany
[3]Instutite for Geometry, TU Dresden, Germany
[1]http://www.tu-cottbus.de/lsebib, [2]http://www.OliverLabs.net,
[3]http://www.DanielLordick.de
[1]gb@hmgb.net, [2]mail@oliverlabs.net, [3]daniel.lordick@tu-dresden.de

Abstract: *We describe the exploration of the manifold novel shapes found in algebraic geometry and their application in architectural design. These surfaces represent the zero-sets of certain polynomials of varying degrees. They are therefore very structured, coherent and harmonious yet at the same time geometrically and topologically highly complex. Their application in design is mostly unprecedended as they have only recently begun to become accessible through novel software tools. We present and discuss experimental student design and research projects where shapes found in algebraic geometry were developed into pavilion designs. We describe historic precedents for the inspiration of art and architecture through mathematics and show how algebraic surfaces can be used to expand architects' sculptural vocabulary, make the utmost of three-dimensional sculptural qualities, employ shapes that have a strong internal structure, transcend the imaginable and explore polynomials as a new kind of shape-making tool.*

Keywords: *Geometry; algebraic geometry; shape; sculpture; design; tool; experiment; methodology; software.*

Motivation and procedure

New interactive and easy to use software tools are triggering a Cambrian explosion of novel shapes which are geometrically and topologically complex but at the same time extremely coherent and harmonious: algebraic surfaces (Figure 1, see www.AlgebraicSurface.net).

We describe an experimental architectural student design project in which pavilion designs were derived from such entities. Our aims are:

Expanding architects' sculptural vocabulary

The larger architects' sculptural vocabulary, the wider the range of designs which can therefore be more adequate, inspiring and humane. The very many diverse and novel shapes of algebraic geometry have not at all been explored yet.

Making the utmost of 3-Dimensional sculptural qualities

The convolutedness and spatial and topological complexity of algebraic surfaces hints at novel

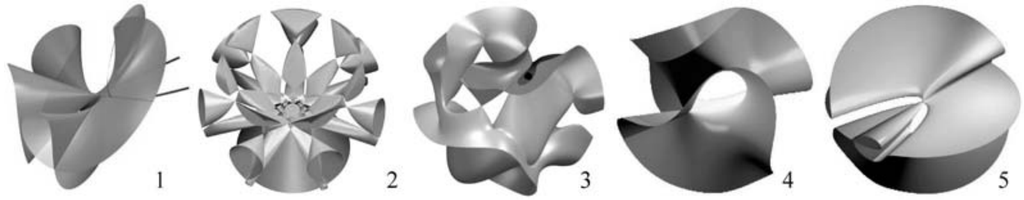

Figure 1
Algebraic surfaces by Oliver Labs (1,2), Herwig Hauser (4,5) and an anonymous creator

possibilities for spatial concepts: manifold internal relationships and multifaceted external appearance.

Using shapes that have a strong internal structure

The sculptural possibilities of CAD software can lead to rather whimsical shapes; a problem not only of aesthetics, but also of engineering and manufacturing logic.
Algebraic surfaces by definition have a very strong internal structure that can avoid such whim and the problems it implies.

Transcending the imaginable

As every new design task essentially requires new and hitherto unknown formulations, designers have to ever expand the boundaries of what they can imagine. Algebraic Geometry does so to an extreme in dealing with entities unimaginable without much training or the aid of calculated visualizations.

Exploring polynomials as a new kind of shape-making tool

New tools extend human capabilities. Visualizing and handling algebraic surfaces via software is a new tool indeed. Its nature and potential so far remains unstudied.

Algebraic surfaces

An algebraic surface V(f) of degree d in three-space $\mathbf{R}^3$ is the set of all points satisfying a certain polynomial equation f of degree d in three unknowns x,y,z:

$$V(f) = \{(x,y,z) \text{ in } \mathbf{R}^3 \text{ with } f(x,y,z) = 0\}$$

First examples are (infinitely large) planes (degree 1) and spheres (degree 2). An algebraic surface may consist of two-dimensional, one-dimensional, and zero-dimensional parts at the same time (Figure 2). They may be symmetric in many ways (the sphere is symmetric with respect to any plane through its centre, the cubic surfaces in figure 2.2+2.3 are rotation-symmetric) and they may even have the symmetries of the classical platonic solids (Figures 2.4, 2.5).

On surfaces in general two sorts of points can be distinguished: over wide areas algebraic surfaces are smooth and consist of regular points with a unique tangent plane each. But often singular points exist, without uniquely definable tangent planes - for example all points which do not look smooth in the figures, e.g. the solitary point in Figure 2.2 or all points which locally resemble the vertex of a cone. Locally at singularities, the shape of a surface may look quite

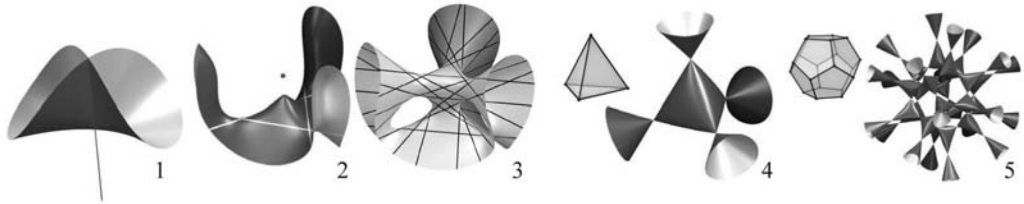

Figure 2
Algebraic surfaces: Whitney Umbrella (1), cubic surfaces (2,3), Cayley Cubic (4), Barth Sextic (5)

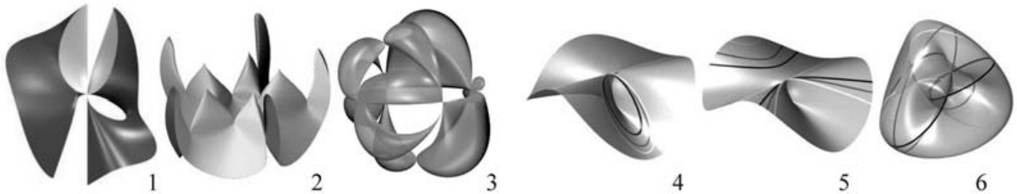

Figure 3
Algebraic surfaces of increasing degree (1-3) and containing conic sections (4-6)

elaborate (Figure 3.1, 3.2 and the referenced surface galleries].

The shapes of algebraic surfaces defined by polynomials of degree <= 3 can be classified: Essentially, all possible shapes of surfaces of degree 2 fit on one sheet of paper [Labs 2008], those of degree 3 are presented in [Holzer, Labs 2006 and Labs 2003]. For surfaces of degree 4, the variety of possible shapes is already far too large to be listed at all although their shapes are quite restricted in many ways. On the other hand, any shape can be approximated up to any desired non-zero precision by an algebraic surface.

Surfaces of lower degrees are easier to handle technically and have more mathematical structure. For example: any cubic surface without singular points contains up to 27 straight lines (Figure 2.3) cut out by 45 planes in triples, and there is a beautiful relationship between the planes and the lines. Furthermore, many surfaces of degree <= 4 contain either a number of lines (Figure 2.3) or a number of conic sections (i.e. circles, ellipses, parabola, etc.) (Figures 3.4, 3.5, 3.6).

We believe that such curves on algebraic surfaces can be viable starting points for architectonic development. These surprising regularities also make the complex entities that contain them more coherent and therefore beautiful.

Mathematics in modern sculpture and architecture

Artists like Man Ray, Naum Gabo, Barbara Hepworth, Max Ernst and Henry Moore (Vierling, 2000), who strove to show and produce objects without natural antetypes, were significantly inspired by mathematical models from the 19th century. Those models illustrated new mathematical entities discovered through the enormous increase of mathematically describable shapes made possible by René Descartes' introduction of algebra into geometry.

Today sculptors like Anish Kapoor (Figure 4.1), Anthony Cragg (Figures 4.2, 4.3) and Eva Hild (Figures 4.4, 4.5) (amongst others) produce sculptures that take cues from mathematical objects (also see http://www.isama.org/hyperseeing/).

The less 'geometric', or, say, cubical a building looks, the more knowledge of geometry will have been required to erect it. Antoni Gaudí, whose work is often misunderstood as an example for a highly idiosyncratic formal language, was a keen student of mathematics, especially geometry. He kept many mathematical models in his studio and often used algebraic surfaces of degree two.

Le Corbusier had Iannis Xenakis developed the Philips Pavilion for the World Expo in Bruxells in 1958 from representations of mathematical entities about which he had previously enquired with mathematicians (Figure 5.1-2) (Treib, 1996).

Several topological ideas have recently been used by architects as organizational metaphors – see for example UN Studio's 'Mobius House' or Mcbride Charles Ryan's 'Klein Bottle House'. Minimal surfaces have been used many times, i.e. in Frei Otto's and Günter Behnisch's Munich Olympic stadium and more recently in Toyo Ito's work (Figure 5.3) and Snøhetta architects' Tubaloon pavilion (Figure 5.4).

Creating and handling algebraic surfaces with computers

The zoo of algebraic surfaces can be explored with several software systems that are very different in

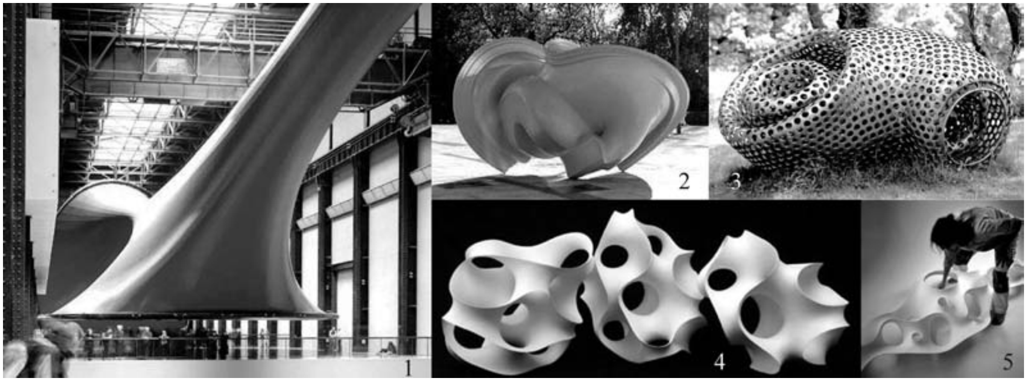

Figure 4
Art works by Anish Kapoor (1), Anthony Cragg (2,3), Eva Hild (4,5)

ease and intuitiveness of use and technical capabilities for exporting the created entities. Surfer, developed for the exhibition Imaginary 2008 (www.imaginary2008.de/), allows easy, free, intuitive and interactive play, but cannot export the created 3D data. 3D-XPlorMath (3d-xplormath.org) provides numerous easily exportable surfaces - but differences between implementations make it difficult to export some user-defined creations. SingSurf can create and export 3D data quickly, but is ill suited for play and experiment. Its export data is also a somewhat rough and not always complete approximation of the true shape. Our final 3D data was thus produced by the EXACUS group in Saarbrücken which develops precise software for relatively simple equations (exacus.mpi-inf.mpg.de). The 3D printer in the 3D LAB B25 at the Dresden University of Technology build our models directly from the CAD data (http://www.math.tu-dresden.de/3D-Labor).

Experimental student project

In an experimental student design project simple exhibition pavilions were derived from algebraic surfaces in several steps (Figures 7, 8, 9):

1. Familiarization with the mathematical concepts of algebraic surfaces and the software that allows to create, modify and export them to CAD packages.
2. Study and discussion of the inspirational relationship between mathematics, art and architecture with a focus on algebraic geometry.
3. Experimental creation of surfaces based on several existing libraries, com-parative discussion of their sculptural qualities, spatial and architectural potential.
4. Creation of a new set of surfaces with more focus on spatial qualities.

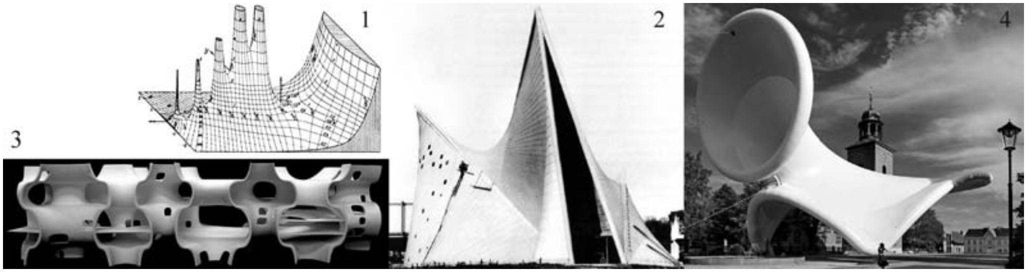

Figure 5
Le Corbusier's and Xenakis' Philips Pavilion (1,2), Ito's Guangzhou (3), Snohetta's Tubaloon (4)

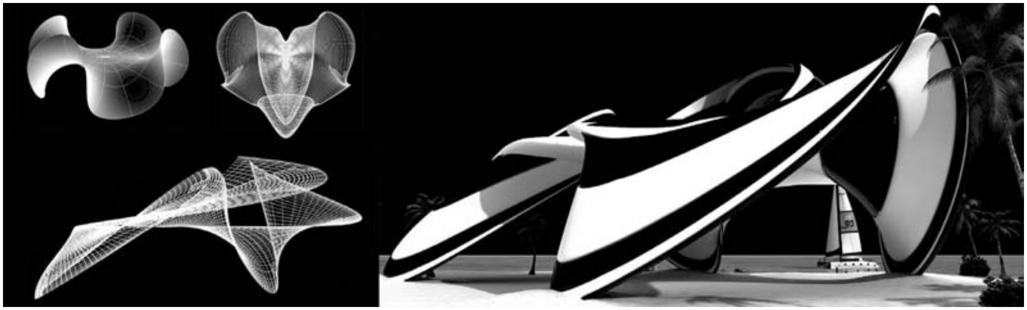

Figure 6
Project derived from the Enneper surface by Marcus Kistner. TU Dresden 2008

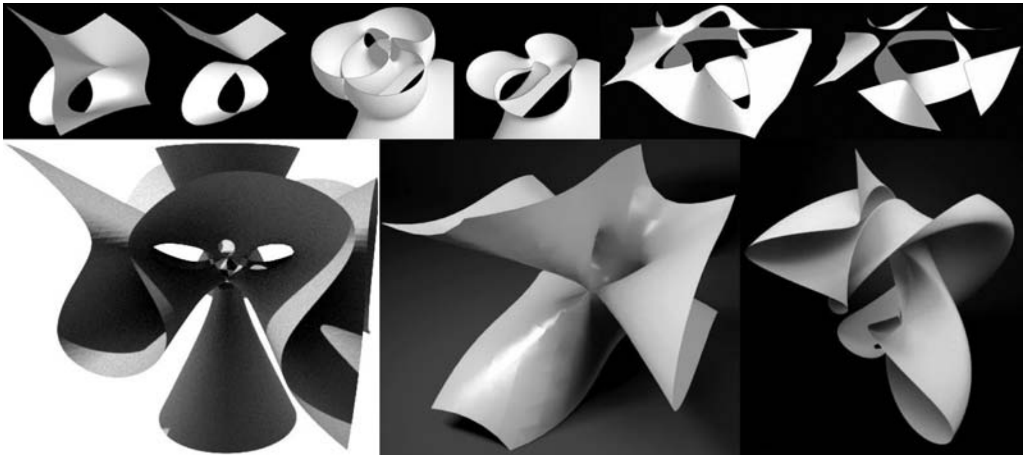

Figure 7
Preliminary design experiments by Torsten Eckert (top row), Stefan Schreck (lower left hand corner) and Jesse Ender, BTU Cottbus 2009

5. Selection of the surfaces with the most potential and development into pavilions by sculptural manipulation in conventional CAD.

To simplify the transition from pure mathematics to concrete architectural design and emphasize the geometric, topological and aesthetic qualities of the surfaces issues of context, function and buildability were minimized. The pavilion designs were modelled in CAD software and shown in plans, sections, perspective renderings and models from 3D prints. To experiment not only with the use of algebraic surfaces, but also with how this can be done with students of architecture, two parallel design projects were conducted at the universities of Cottbus and Dresden. For a more tactile access, the students in Dresden produced cardboard models of the shapes by generating a rib structure with a laser cutter. Nontrivial mathematical surfaces were used in design courses before, although generally restricted to adopting existing surfaces (Figure 6 and Maertterer, Saunders)

Evaluation

The mathematical technicalities and spatial complexity of algebraic surfaces challenged the students due, we believe, to shortcomings in high school education, but Surfer's playfulness helped to overcome this. Moreover, the technical roundabouts (see 4.) restricted the students' creativity. They furthermore

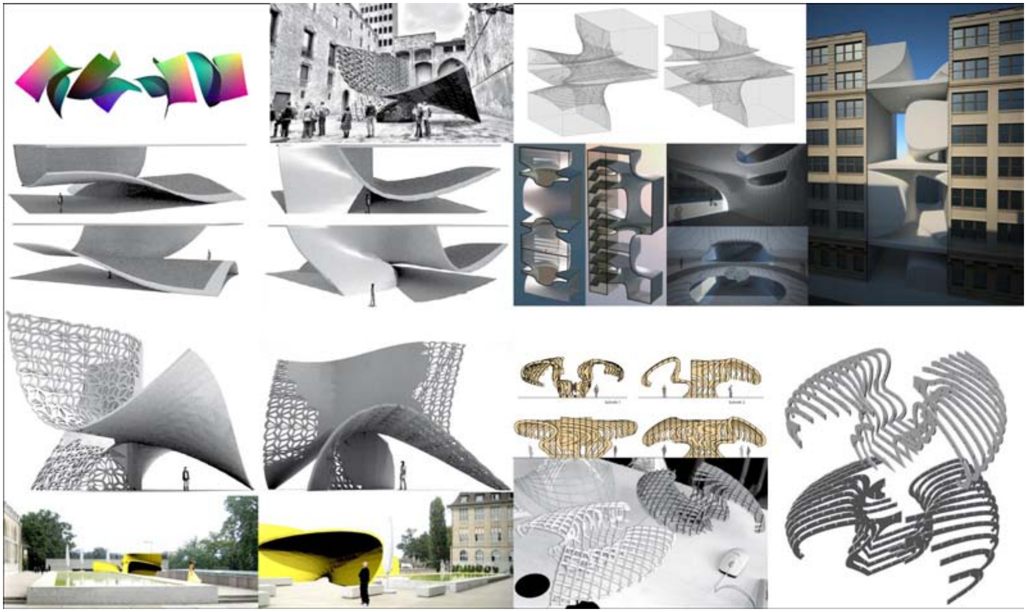

Figure 8
Experimental student pavilion designs (clockwise from lower left hand corner): Tobias Hesse, Stefan Schreck, Christopher Jarchow, BTU Cottbus 2009, Michael Hasse, TU Dresden 2009

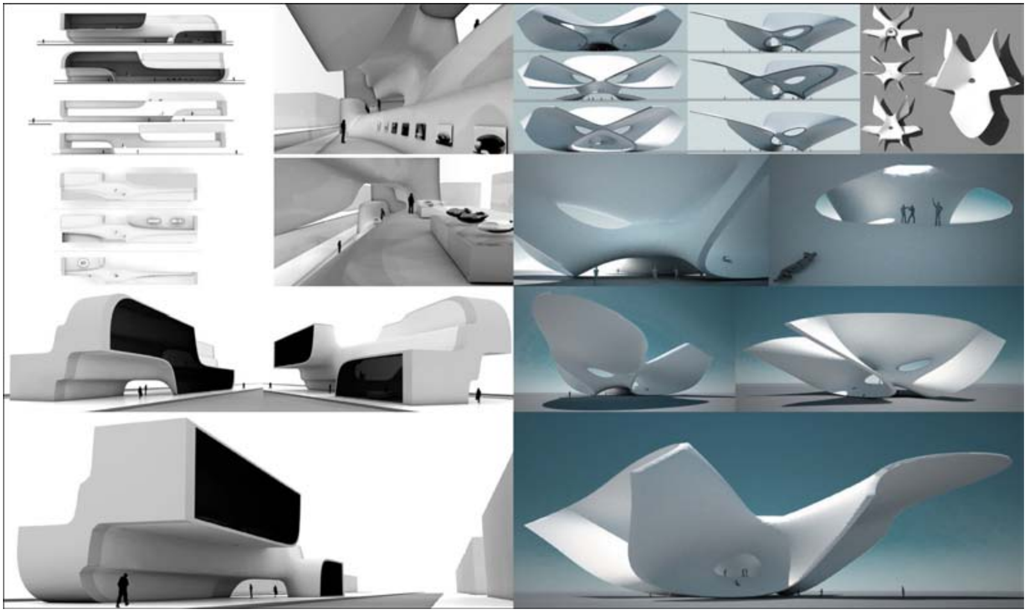

Figure 9
Experimental student pavilion designs: Tony Jacobs (left) & Xing Jiang (right),BTU Cottbus 2009

found it difficult to comprehend the sculptural complexity of the algebraic surfaces and to envision architectural applications for them.

This was partly overcome by repeatedly visualizing and handling many different surfaces interactively in the mathematical and CAD software and thereby over time becoming familiar with them. The students also had to invert their usual design process where envisioning precedes visualization. Here discovery and development of architectonic potential succeeded visualization.

An uncommonly high degree of serendipitous experimentation was required.

Still, the students greatly expanded their sculptural vocabulary and their skills in interpreting and visualizing complex three-dimensional structures. The resulting pavilion designs greatly exceeded what the students had been able to imagine, handle and design before. The involvement with mathematical structures, parameters and even with the limitations in the software lead to deep insights into spatial concepts for the management of complexity beyond basic solids.

Possibilities for further investigations

More exploration of the 'zoo of surfaces'

Further investigation and experimentation beyond the area currently explored in algebraic surfaces created for aesthetic reasons promises to open up many more spatial possibilities for architecture.

The sculptors of constructivism were inspired by mathematical models, but neither comprehended them nor communicated with their creators and were therefore unable to really integrate them into their artistic endeavour. Today mathematicians do not only provide models but more importantly the means for creating and understanding them, and they can be easily contacted for discussion.

Also, the original mathematical artefacts' aesthetic value was merely a side effect of scientific study and teaching. Many more surfaces than the mathematically interesting ones exist. They often have more aesthetic qualities and can now be explored by the mathematically untrained with the new software tools described above. This active exploration is significantly different from the constructivist artists' passive consumption. For lack of exploratory means and therefore vision they mistook for a dead-end street the route to great discoveries.

Transforming algebraic surfaces into building structures

Going beyond 3D print models to construct larger scale models or 1:1 objects would require more development of the geometry (especially segmentation and tessellation) than possible within the time limits of the courses. A first attempt was to create rib structures in the Dresden course; a standard solution borrowed from ship design that makes severe aesthetical and practical problems evident. Technically and aesthetically convincing strategies to transform nontrivial algebraic surfaces into building structures should be developed.

Accommodating complex architectonical functional spatial programs

The shapes of algebraic geometry hint at new possibilities for the spatial organization of functional programmes more complex than the simple ones covered by us. Understanding and using them requires further study and experiment.

Building a library of surfaces

A library of the currently unfamiliar forms of algebraic geometry, sorted and accessible with different tags like for example degree of generating polynomial, number of self-intersections, enclosures and tunnels and number and types of singularities together with animated renderings, smooth high-resolution 3D models and explanations of the underlying mathematics would greatly facilitate grasping their potential - even if it would necessarily be always incomplete as the shapes generated by polynomials of higher degrees cannot be classified so far.

Systematically listing and analyzing how art and architecture were inspired by mathematical discoveries

A thorough study of the numerous inspirations artists and architects drew from mathematics would broaden designers' view, improve their understanding, thereby provide the basis for further inspiration and energize creative development.

Development of software tools

Replacing the current cumbersome production pipeline of different software systems with Plug-in tools for existing CAD software that have the intuitiveness and functionality of Surfer combined with an ever-growing surface library would accelerate experimental feedback loops and drive design capabilities forward.

Study of polynomials as a novel shape-making tool and how the creators of surfaces that serve beauty not reason use it

Obtaining surfaces from polynomials via software is so far - like any new tool - rather complicated, not very intuitive and its creative potential mostly restricted to those with much knowledge or time for trial and error or both. Studying the already beautiful and novel results of the Imaginary2008 exhibition and the design competitions of 'Spektrum der Wissenschaft' and 'Zeit Wissen' and their creators' working methods would improve understanding of the new tool.

The aesthetics of algebraic surfaces: Beauty through restriction

At least since the mathematician Felix Klein presented models of algebraic surfaces at the World's Fair at Chicago in 1893 many non–mathematicians became fascinated by their beauty and inherent structure. Today, those are even more apparent through computerized visualization and model-making. Over 120,000 people saw the examples displayed in Imaginary2008 exhibition touring through Germany in 2008. Still, the visual appeal and fascination especially for many non-specialists who lack technical understanding so far remains unstudied.

Algebraic surfaces appear to be more beautiful the lower the degree of the producing polynomial is (if it is higher than 2) (Figure 3.1-3). The higher the degree, the more arbitrary the surfaces. Their beauty partly certainly lies in the surprising balance between convoluted complexity and harmonic consistency.

Such balance between economy of production and beauty of result is as a matter of course common to all creative endeavour, but understanding its workings related to algebraic surfaces would help to release their creative potential.

Acknowledgements

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References

Brüderlin, M. [Ed.]: 2004, ArchiSculpture, Exhibition Catalogue.

Burry, M. and others: 2007, Gaudí Unseen, Catalogue of the exhibition in the Deutsche Architekturmuseum.

Hilbert, D. and Cohn-Vossen, S.: 1952, Geometry and the Imagination, Chelsea Publishing Company.

Holzer, S. and Labs, O. in: Elkadi, M. and Mourrain, B. and Piene, R.: 2006, Algebraic Geometry and Geometric Modeling, ch. Illustrating the Classification of Real Cubic Surfaces, Springer.

Hupasch, V. and Lordick, D.: 2008, Good Vibrations - Geometrie und Kunst, Exhibition Catalogue, Dresden.

Ito, T. in: 2005, Architecture + Urbanism #417 'Toyo Ito / Beyond the Image', Toyko.

Labs, O.: 2003, Algebraic Surface Homepage. Information, Images, Tools, www.AlgebraicSurface.net.

Labs, O.: 2008, Weltrekordflächen, www.Imaginary2008.de.

Labs, O.: 2009, A List of Challenges for Real Algebraic Surface Visualization Software, in preparation.

Mehrtens, H.: Mathematical Models. In: 2004, Models. the third dimension of science. Edited by Soraya de Chadarevian and Nick Hopwood; Stanford, California, pp. 276 – 306.

Meyer, H., Stussak, C., O. Labs and A. Matt: 2008, surfer – Visualization of Real Algebraic Surfaces, www.imaginary2008.de..

Treib, M.: 1996, Space calculated in seconds, Princeton.

Vierling-Claassen, A.: 2000, Mathematical Models and Art in the Early 20th Century, http://www.math.harvard.edu/~angelavc/models/index.html.

Harris, E.: 2009, 'Surfaces 1+2', http://maxwelldemon.wordpress.com/2009/03/21/surfaces-1-the-ooze-of-the-past/, Blog Entry 2009.

Hyperseeing Magazine: http://www.isama.org/hyperseeing/

Maertterer, J. and Saunders, A.: Architecture and the Rhino Math Plug-In, http://de-de.de/rhino3/album/math/index.shtml

Mcbride/Charles/Ryan: Klein Bottle House: http://www.mcbridecharlesryan.com.au

Minifie Nixon Architects: Australian Wildlife Center, http://www.minifienixon.com

UN studio: Mobius House, http://www.unstudio.com/projects/name/M/1/118

http://www.imaginary2008.de/

http://kommentare.zeit.de/article/2008/01/23/mathematik-skulpturenwettbewerb

http://www.spektrum.de/page/p_sdwv_mathekunst&_z=798888

http://www.freigeist.cc/gallery.html

http://www.singsurf.org/

http://www.math.tu-dresden.de/3D-Labor

http://www.math.tu-dresden.de/modellsammlung/

http://www.rhino3.de/_develop/__v3_plugins/math/index.shtml

www.altana-galerie-dresden.de/ausstellungen/good-vibrations/start